



# PhysCon2013

## *Hamiltonian Systems and optimal control*

**Schedule:** We-am1, 9:00- 10:00hrs

**Auditorium:** "Rafael Nieto" Rectory Building UASLP



**Speaker:** Professor Velimir Jurdjevic

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### **Abstract:**

This lecture will highlight the contributions of optimal control theory to geometry and mechanics. The basic object of study is the reachable sets of families of vector fields parametrized by control functions. We will show how the extremal properties of the reachable sets lead to the Hamiltonians and how these Hamiltonians alter our understanding of the classical calculus of variations in which the Euler-Lagrange equation is a focal point of the subject.

The second part of the lecture will be focused on systems with symmetries, integrals of motion and integrability. We will show the relevance of symplectic geometry and the associated Hamiltonian formalism on Lie groups  $G$  whose Lie algebras  $\mathfrak{g}$  admit Cartan decomposition  $\mathfrak{g} = \mathfrak{p} + \mathfrak{k}$  subject to Lie bracket relations

$$[\mathfrak{p}, \mathfrak{p}] \subseteq \mathfrak{k}, [\mathfrak{p}, \mathfrak{k}] \subseteq \mathfrak{p}, [\mathfrak{k}, \mathfrak{k}] \subseteq \mathfrak{k}$$

by singling out certain left-invariant optimal control problems whose solutions give new insights into the geometry of the underlying Riemannian symmetric spaces and also clarify certain aspects of the theory of integrable Hamiltonian systems.

**Short CV:****1. Degrees:**

2005-present Professor Emeritus, University of Toronto  
1971-2005 Professor, Mathematics, University of Toronto  
1970-71 Post-Doctoral Fellow, Harvard University, Cambridge, Mass.  
1969 Ph.D. Case Western Reserve University, Cleveland, Ohio.

**2. Other Teaching and Research Experience**

Spring 2012 Visiting Member of the Institute Mittag-Leffler, Stockholm, Sweden  
Summer 2010 Visiting Lecturer, Summer School in Geometria, Control Y Aplicaciones  
Universidad de Los Andes, Colombia  
Spring 2009 Visiting Researcher, Kavli Institute for Theoretical Physics Department of Physics,  
University of California at Santa Barbara  
Summer 2009 Lecturer, Summer School in Geometry, Mechanics and Control Theory,  
University of Barcelona, Spain,  
Spring 2003 Visiting Member of the Institute Mittag-Leffler, Stockholm, Sweden  
1997-1998 Visiting Professor, University of Florence, Italy  
Visiting Professor, Institute des Sciences Appliquées, Rouen, France  
1993-1994 Visiting Professor, Mathematics, University of Bourgogne April  
Member of the Institute for Advanced Study, Princeton, New Jersey  
Visiting Professor, Mathematics, Steklov Institute of Mathematics, Moscow.

**3. Invited Lectures**

2012 Integrable Hamiltonian Systems on Symmetric Spaces  
Conference on Geometric Control theory, Dijon, France.  
2012 Jacobi's geodesic problem and Lie groups  
Geometric Control and sub-Riemannian Geometry, Corona, Italy.  
2011 Optimal Control in Geometry and Mechanics  
Conference on Geometry, Dynamics and Integrable systems, Sintra, Portugal.  
2010 Optimal Control Problems on Lie groups and Integrable Systems,  
Conference on Geometry, Dynamics and Integrable Systems, Sept 7-13, 2010, Belgrade, Serbia  
2010 Optimal Control on Lie Groups and their homogeneous spaces,  
Queens University, Kingston, Ontario  
2009 Integrable Hamiltonian Systems on Lie groups and Symmetric Spaces,  
Canad. Math. Soc. Meeting, Windsor, Ont  
2009 Integrable Hamiltonian Systems, University of Loughborough, U.K.  
2007 Optimal Control Problems, Mechanics and Geometry, plenary lecture  
Joint Meeting AMS and the Brazilian Math. Soc., Rio de Janeiro, Brazil.  
2003 S. Kowalewski: The Passion, The Mystery and the Genius -  
Mittag - Leffler Institute, Stockholm, Sweden  
The Elastic Curves, Tops and the Optimal Control,  
Mittag-Leffler Institute, Stockholm, Sweden.

**4. Books**

1. Geometric Control Theory, Advanced Studies in Mathematics, Vol. 52, Cambridge University Press.
2. Geometric Control and Non-Holonomic Problems in Mechanics, Editor jointly with R. Sharpe, Canad. Math. Soc. Conference Proceedings, 1998.
3. Integrable Hamiltonian Systems on Complex Lie Groups, Memoirs Amer. Math. Soc., Vol 178, No 838, 2005.

5. Select list of publications

1. Integrable Hamiltonian Systems on Lie Groups: Kowalewski type, *Annals of Mathematics* 150, 1999, p. 605-644.
2. On Removing Barriers between Mechanics and Optimal Control: Completely Integrable Systems, *Proceedings of 37 th IEEE Conference on Decision and Control, Tampa, Florida (1998)*, pp. 2244–2250.
3. Complex Hamiltonians and Integrable Systems, *Proceedings of 38th Conference on Decision and Control, Phoenix, Arizona (1999)*.
4. Hamiltonian Point of View of Non-Euclidean Geometry and Elliptic Functions, *Lie Theory and Application in Control Systems*, edited by U. Helmke, K. Hueper and J. Lawson, *System and Control Letters* 43 (2001), 25–41.
5. with F. Monroy-Pérez, Variational Problems on Lie Groups and their Homogeneous spaces: Elastic Curves, Tops and Constrained Geodesic Problems, *Contemporary Trends in Control Theory and Applications World, Scientific Publishing, Feb 2002* edited by B. Bonnard, J.P. Gauthier and F. Monroy-Pérez
6. Integrable Hamiltonian Systems on Complex Lie Groups, *Memoirs Amer. Math. Soc.*, Vol 178, No 838, 2005, 1-133.
7. V. Jurdjevic and J. Zimmerman, Rolling Sphere Problems on Spaces of Constant Curvature, *Math. Proc. Camb. Phil. Soc.* (144), 2008, 729-747.
8. V. Jurdjevic, The Maximum Principle in Optimal Control, in *Encyclopedia of Complexity and Systems Science* (edited by R.A. Meyers), (2009), Part 13, pp. 5440-5450.
9. V. Jurdjevic, The symplectic structure of curves in three dimensional spaces of constant curvature and the equations of mathematical physics, *Ann. I. H. Poincare* (26), 2009, 1483-1515.
10. V. Jurdjevic, Optimal control systems on Lie groups and integrable Hamiltonian Systems *Jour. of Regular and Chaotic Dynamics* Vol 16, No. 5 (2011), pp. 514-545.
11. V. Jurdjevic, Elastic Problems and Optimal Control: Integrable systems, to appear in *Zbornik Radova, Serbian Academy of Sciences*.
12. V. Jurdjevic, Affine-Quadratic problem on Lie groups, to appear in the *Mathematical Control and related fields* in the special issue dedicated to B. Bonnard on his 60<sup>th</sup> Birthday.
13. V. Jurdjevic, Second-order Systems and the Hybrid Maximum Principle, to appear in the special issue dedicated to A. Agrachev on the occasion of his 60th Birthday, *Springer INDAM Series*.